Pay-As-You-Go Public Pension Systems: Two-Sided Altruism and Endogenous Growth

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Abstract

Within the framework of an overlapping generations model with two-sided altruism and endogenous growth, this paper calculates the rates of fertility, output growth, child-rearing cost, saving, consumption, net intertemporal transfer, bequest and gift, and compares the equilibrium solutions under different public pension systems. It proves that the fully-fertility-linked public pension system (FFLPPS) is equivalent to the system without public pension (WPPS), and the partly-fertility-linked public pension system (PFLPPS) is equivalent to the conventional public pension system (CPPS). The CPPS is beneficial to developing countries in promoting economic growth and reducing population. It is necessary for developed countries to weigh gains and losses carefully if they hope to transform their CPPS (or PFLPPS) to the FFLPPS.

I. Introduction

Some developing countries are suffering from low economic growth and over-population, while developed countries are suffering from low fertility and population aging. It is argued that there is an interrelation between the population problem and the public pension system, and economic growth is impacted by the public pension system.

Nishimura and Zhang (1995) use an exogenous model to compare a system without public pension, a totally fertility-dependent public pension system, and a conventional or partly fertility-dependent public pension system. Adopting the conclusion of Nishimura and Zhang (1992)—the optimal allocation with public pension is not sustainable when fertility is endogenous, they assume that individuals take the public pension level set by the government as given to maximize their own utility. Assuming that altruism runs from children to parents, they calculate fertility, gifts, savings, work-period consumption and utility. One of their results, the introduction of the totally fertility-dependent public pension system[26] reduces the fertility rate, seems doubtful intuitively. It is also theoretically contradictory to the result of Groezen *et al.* (2003).

Zhang and Zhang (1995) extend the model of Nishimura and Zhang (1995). They use an endogenous growth model to compare the system without public pension, the fully fertility-related public pension system, and the conventional public pension system. Assuming that altruism runs from children to parents, they examine the effects of the different public pension systems on the rates of fertility and output growth.

This paper extends Nishimura and Zhang (1995) and Zhang and Zhang (1995). It uses an endogenous growth model to compare the system without public pension (WPPS), the conventional public pension system (CPPS), the fully-fertility-linked public pension system (FFLPPS) and the partly-fertility-linked public pension system (PFLPPS). It assumes that altruism is two-sided: children give gifts to parents and parents leave bequests to children. It calculates not only the rates of fertility, gift, saving, consumption, but also the rates of output growth, bequest, net intertemporal transfer and child-rearing cost.

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[26] The pension benefits are strictly linear increasing with individual's children number. See Nishimura and Zhang (1995) for details.

Bequests are sizeable wealth in real life. Kotlikoff and Summers (1981) report that 80% of U.S. household wealth is inherited wealth. Abel and Warshawsky (1988) and Zhang and Zhang (2001) classify bequest motives into altruistic, exchange, joy-of-giving and accidental motives. In fact, it is hard for most people to distinguish bequest motives clearly when they leave bequests to their children. Abel and Warshawsky (1988) analyze the joy-of-giving bequest motive in which the utility obtained from leaving bequests depends only on the size of the bequests. They exploit the fact that this formulation can be interpreted as a reduced form of the altruistic motive for most purposes. Zhang and Zhang (2001) prove that altruistic and exchange motives yield equivalent outcomes if the discount factors are set the same. Sheshinski and Weiss (1981) consider accidental bequests as a determinant of the representative individual's utility function. This paper also takes bequests as a determinant of the representative individual's utility function.

Following Saint-Paul (1992), Zhang and Zhang (1995, 1998, 2001), Wigger (1999a, 1999b) and so on, this paper adopts the endogenous growth model with Romer's (1986) type of capital externality. It is tractable to compare the balanced growth equilibrium solutions of the system without public pension (WPPS), the conventional public pension system (CPPS), the fully-fertility-linked public pension system (FFLPPS) and the partly-fertility-linked public pension system (PFLPPS).

Within the framework of an overlapping generations model with two-sided altruism and endogenous growth, this paper obtains some interesting results that have not appeared in the literature. It proves that the rates of fertility, output growth, child-rearing cost, saving, consumption, net intertemporal transfer, bequest and gift under the FFLPPS are the same as those under the WPPS. The rates of fertility, output growth, child-rearing cost, saving, consumption, net intertemporal transfer, bequest and gift under the PFLPPS are the same as those under the CPPS. The FFLPPS is equivalent to the WPPS, and the PFLPPS is equivalent to the CPPS.

The rates of fertility and child-rearing cost under the CPPS (or PFLPPS) are smaller than those under the FFLPPS (or WPPS).

The rates of output growth, consumption, net intertemporal transfer, bequest and gift under the CPPS (or PFLPPS) are greater than those under the FFLPPS (or WPPS).

The rate of net intertemporal transfer under the WPPS is larger than the rate of net intertemporal transfer outside of the CPPS, both of which are positive. However, the rate of net intertemporal transfer outside of the PFLPPS is zero, and the rate of net intertemporal transfer outside of the FFLPPS is negative.

The gift rate under the WPPS is the largest. The gift rate outside of the public pension system under the CPPS is larger than that under the PFLPPS, which in turn is larger than that under the FFLPPS.

Section 2 presents the basic model. Section 3 computes the stationary optimal allocation for choosing a rational pension level. Sections 4, 5 and 6 compute the equilibrium solutions under the CPPS, the FFLPPS and the PFLPPS. Section 7 compares the equilibrium solutions. Section 8 concludes the paper.

II. The Basic Model: They System without Public Pension (WPPS)

Individuals. Individuals live for two periods. The generation born at the beginning of period *t* is called generation *t*. Each individual of generation *t* earns wage income by supplying inelastically one unit of labor, receives bequest income from his parent, consumes part of his incomes, rears his children, gives his parent gifts, and saves the rest of his incomes in his work period or first period. In his retirement period or second period, he consumes part of the sum of his savings and gifts received from his children, and leaves the remaining part as bequests to his children.

Each individual of generation *t* derives utility from his work-period consumption C_t^t, his retirement-period consumption C_{t+1}^t, his bequest left to each of his children B_{t+1} and the retirement-period consumption of his parent C_t^{t-1}. The representative individual maximizes his utility by choosing the rates of saving, gift, fertility in work period and the rate of bequest in retirement period:

$$Max_{\{s_t, g_t, n_t, b_{t+1}\}} U(C_t^t, C_{t+1}^t, C_t^{t-1}, B_{t+1}) = \alpha \ln C_t^t + \beta \ln C_{t+1}^t + \gamma \ln C_t^{t-1} + \sigma \ln B_{t+1}, \qquad (1)$$

$$s.t. \quad C_t^t = W_t + b_t W_t - (s_t + g_t + h_t) W_t, \qquad (2)$$

$$C_{t+1}^t + (1+n_t) b_{t+1} W_{t+1} = (1+r_{t+1}) s_t W_t + (1+n_t) g_{t+1} W_{t+1}, \qquad (3)$$

where W_t denotes the wage, s_t the saving rate, g_t the gift rate, and r_{t+1} the interest rate.

$$B_{t+1} = b_{t+1} W_{t+1}, \qquad (4)$$

where b_{t+1} denotes the ratio of the bequest to the wage per worker of generation t+1[27], which is called the bequest rate for simplification. The child-rearing cost rate is assumed to be:

$$h_t = q(1+n_t)^d, \qquad (5)$$

where n_t is the fertility rate, $q > 0, d \geq 1$, such that the costs of rearing children are either linear or convex. Let $C_t^t = c_{1t} W_t$, where c_{1t} denotes the rate of work-period consumption. Assume that $s_t, g_t, n_t, b_{t+1} > 0$ for all t throughout this paper to focus on interior solution. The weights in the utility function are assumed as such that $\alpha > \beta > \gamma > \sigma > 0$, $\alpha + \beta + \gamma + \sigma = 1$.

Firms. Firms produce a single commodity in competitive markets. The production function $Y_t = F(K_t, A_t L_t) = A_t L_t \cdot f(k_t)$ is homogeneous of degree one, where Y_t denotes output in period t, K_t capital stock, A_t labor productivity, L_t labor, $k_t = K_t/(A_t L_t)$ capital per unit of effective labor. Euler's theorem gives:

$$r_t = f'(k_t), \qquad (6)$$

$$w_t = W_t / A_t = f(k_t) - k_t f'(k_t), \qquad (7)$$

where w_t is the wage rate per unit of effective labor. In order to ensure the existence of a balanced growth path for the economy, the following particular form of A_t is adopted (see, e.g., Saint-Paul, 1992; Zhang and Zhang, 1995, 1998; Wigger, 1999a, 1999b; etc.):

$$A_t = K_t / (a L_t), \qquad (8)$$

where a is a positive technological parameter.

Therefore $k_t = a$, and

$$r_t = f'(a) = r, \; w_t = f(a) - af'(a) = w \text{ for all } t. \qquad (9)$$

The Goods Market. The goods market equilibrium requires that the demand for goods in each period be equal to the supply:

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[27] Following Veall (1986) and Nishimura and Zhang (1995), this paper only considers Nash equilibrium. That is, each individual takes the decisions of future generations as given.

$$K_{t+1} = S_t = s_t W_t L_t\ .^{28} \tag{10}$$

The Equilibrium Solutions. Combining equations (7), (8), (9), (10) and the labor force $L_{t+1} = (1+n_t)L_t$ yields the growth rate of capital per worker:

$$1+\theta_t = (K_{t+1} / L_{t+1})/(K_t / L_t) = s_t w /[a(1+n_t)]\ . \tag{11}$$

Substituting equation (8) into the production function and using $k_t = a$ gives that the growth rate of output per worker is equal to the growth rate of capital per worker. From equations (7) and (9), one can get the growth rate of wage:

$$W_{t+1} / W_t = 1+\theta_t\ . \tag{12}$$

A balanced growth equilibrium is a competitive equilibrium in which the saving rate, the gift rate, the fertility rate, etc., are constant, but the wage, the work-period consumption C_t^t, the retirement-period consumption C_{t+1}^t, etc., grow at the same endogenously determined and constant growth rate of capital per worker.

Lagging equation (3) and substituting equations (2)-(5) into equation (1), differentiating equation (1) with respect to s_t, g_t, n_t, b_{t+1} gives the first-order conditions for the maximization problem of equations (1)-(3):

$$\alpha / C_t^t = (1+r)\beta / C_{t+1}^t\ , \tag{13}$$

$$\alpha / C_t^t = (1+n_{t-1})\gamma / C_t^{t-1}\ , \tag{14}$$

$$\left(dh_t W_t / (1+n_t)\right)\left(\alpha / C_t^t\right) = (g_{t+1} - b_{t+1}) W_{t+1} \beta / C_{t+1}^t\ , \tag{15}$$

$$(1+n_t)\beta / C_{t+1}^t = \sigma / B_{t+1}\ . \tag{16}$$

Equation (13) means the tradeoff between the marginal utility of work-period consumption and that of retirement-period consumption through savings. Equation (14) means the tradeoff between the marginal utility of the representative individual's work-period consumption and that of his parent's retirement-period consumption through gifts. Equation (15) means the tradeoff between the marginal utility of child-rearing costs and that of net transfers from children to parent. Equation (16) means the tradeoff between the marginal utility of retirement-period consumption and that of bequests left to children.

Equating equations (13) and (14), inserting equation (3) and using equation (12) yields:

$$(1+\bar{n})(1+\bar{\theta}) = (1+r)\beta / \gamma\ . \tag{17}$$

Dividing equation (15) by equation (13) yields:

$$\bar{\tau} = d\bar{h}\gamma / \beta\ , \tag{18}$$

where $\bar{\tau} = \bar{g} - \bar{b}$ is the rate of net intertemporal transfer in equilibrium.

Substituting equation (17) into equation (11) yields:

[28] See Blanchard and Fischer (1989, p.94) or Barro and Sala-I-Martin (1995, p.130) for details.

$$\bar{s} = \beta e / \gamma , \quad (19)$$

where $e = a(1+r)/w$.

Substituting equations (2) and (3) into equation (13) yields:

$$\bar{h} = [1 - e(\alpha + \beta)/\gamma]/[1 + d(\alpha + \gamma)/\beta] . \quad (20)$$

Substituting equations (18), (19) and (20) into equation (2) yields:

$$\bar{c}_1 = 1 - \beta e / \gamma - (1 + d\gamma / \beta)\bar{h} . \quad (21)$$

Substituting equation (3) into equation (16), and using equation (12) yields:

$$\bar{b} = \sigma(e + d\bar{h}\gamma / \beta)/\beta , \quad (22)$$

$$\bar{g} = d\bar{h}\gamma / \beta + \bar{b} . \quad (23)$$

III. The Stationary Optimal Allocation

The government maximizes the balanced growth welfare[29] by choosing the stationary optimal rates of saving, gift, fertility and bequest, and sets a rational pension level according to the optimal rate of net intertemporal transfer. Therefore the government solves the following maximization problem:

$$Max_{\{s,g,n,b\}} U_t = \alpha \ln C_t^t + \beta \ln C_{t+1}^t + \gamma \ln C_t^{t-1} + \sigma \ln(bW_{t+1}), \quad (24)$$

$$s.t. \quad C_t^t = (1 + b - s - g - h)W_t , \quad (25)$$

$$C_{t+1}^t = (1+r)sW_t + (1+n)(g-b)W_{t+1} . \quad (26)$$

Manipulation analogous to section 2 gives the optimal solutions:

$$(1+n_p)(1+\theta_p) = 1 + r , \quad (27)$$

$$\tau_p = dh_p , \quad (28)$$

$$s_p = e , \quad (29)$$

$$h_p = [1 - e(\alpha + \beta + \gamma)/(\beta + \gamma)]/[1 + d(\alpha + \beta + \gamma)/(\beta + \gamma)] , \quad (30)$$

$$c_p = 1 - e - (1+d)h_p . \quad (31)$$

Therefore the government sets the PAYG public pension tax rate as τ_p .

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[29] This utility function can be considered as a social welfare function. See Nishimura and Zhang (1995, fn.10) and Zhang and Zhang (1995, p.444) for details.

IV. The Conventional Public Pension System (CPPS)

Under the pay-as-you-go (PAYG) public pension system, the government levies pension tax $\tau_p W_t$ on each worker in period *t*, and pays pension benefits to each retiree in the same period.

The pension benefits are related to the average social fertility rate under the CPPS. Taking the public pension level as given, each individual may give extra gifts to his parent, leave extra bequests to his children and choose the other variables deviating from the optimal allocation. Each individual maximizes his utility by choosing the rates of saving, gift, fertility and bequest. Thus the representative individual solves the following maximization problem: (1) subject to:

$$C_t^t = W_t + b_t W_t - [s_t + \tau_p + g_t + h_t] W_t , \tag{32}$$

$$C_{t+1}^t + (1+n_t) b_{t+1} W_{t+1} = (1+r_{t+1}) s_t W_t + (1+n_{at}) \tau_p W_{t+1} + (1+n_t) g_{t+1} W_{t+1} , \tag{33}$$

where n_{at} is the average social fertility rate and is equal to n_t in equilibrium.

Manipulation analogous to section 2 gives the equilibrium solutions:

$$(1+\tilde{n})(1+\tilde{\theta}) = (1+r)\beta / \gamma , \tag{34}$$

$$\tilde{s} = \beta e / \gamma , \tag{35}$$

$$\tilde{h} = [1 - e(\alpha+\beta)/\gamma - dh_p(\alpha+\gamma)/\gamma] / [1 + d(\alpha+\gamma)/\beta] , \tag{36}$$

$$\tilde{c}_1 = 1 - \beta e / \gamma - dh_p - (1 + d\gamma/\beta)\tilde{h} , \tag{37}$$

$$\tilde{\tau} = d\tilde{h}\gamma / \beta , \tag{38}$$

$$\tilde{b} = \sigma[e + dh_p + d\tilde{h}\gamma/\beta)] / \beta , \tag{39}$$

$$\tilde{g} = d\tilde{h}\gamma/\beta + \tilde{b} , \tag{40}$$

where $\tilde{\tau}$ is the rate of net intertemporal transfer excess of τ_p, or the rate of net intertemporal transfer outside of the CPPS. The total rate of net intertemporal transfer in the equilibrium of the CPPS is $\tau_p + \tilde{\tau}$. And $\tau_p + \tilde{g}$ is equivalent to the total gift rate in the equilibrium of the CPPS.

V. The Fully-fertility-linked Public Pension Systen (FFLPPS)

The public pension benefits are completely dependent on individual fertility rate under the FFLPPS. Taking the public pension level as given, each individual additionally gives his parent gifts during his work period and leaves bequests to his children during the retirement period. The net intertemporal transfers outside of the FFLPPS depend on the following computation instead of discussions such as Nishimura and Zhang (1995) and Zhang and Zhang (1995). Each individual maximizes his utility by choosing the rates of saving, gift, fertility and bequest. The representative individual solves the following maximization problem: (1) subject to:

$$C_t^t = W_t + b_t W_t - [s_t + \tau_p + g_t + h_t] W_t , \tag{41}$$

$$C_{t+1}^{t} + (1+n_t)b_{t+1}W_{t+1} = (1+r_{t+1})s_tW_t + (1+n_t)\tau_pW_{t+1} + (1+n_t)g_{t+1}W_{t+1}. \quad (42)$$

The equilibrium solutions are:

$$(1+\hat{n})(1+\hat{\theta}) = (1+r)\beta/\gamma, \quad (43)$$

$$\hat{s} = \beta e/\gamma, \quad (44)$$

$$\hat{h} = [1-e(\alpha+\beta)/\gamma]/[1+d(\alpha+\gamma)/\beta], \quad (45)$$

$$\hat{c}_1 = 1-\beta e/\gamma-(1+d\gamma/\beta)\hat{h}, \quad (46)$$

$$\hat{\tau} = d(\hat{h}\gamma/\beta-h_p), \quad (47)$$

$$\hat{b} = \sigma(e+d\hat{h}\gamma/\beta)/\beta, \quad (48)$$

$$\hat{g} = d(\hat{h}\gamma/\beta-h_p)+\hat{b}, \quad (49)$$

where $\hat{\tau}$ is the rate of net intertemporal transfer excess of τ_p, or the rate of net intertemporal transfer outside of the FFLPPS. The total rate of net intertemporal transfer in the equilibrium of the FFLPPS is $\tau_p+\hat{\tau}$. And $\tau_p+\hat{g}$ is equivalent to the total gift rate in the equilibrium of the FFLPPS.

VI. The Partly-fertility-linked Public Pension System (PFLPPS)

Under the PFLPPS, the pension benefits include two parts: one is related to the average social fertility rate; another one is dependent on the individual fertility rate. Taking the public pension level as given, each individual additionally gives his parent gifts and leaves bequests to his children. The net intertemporal transfers outside of the PFLPPS depend on the following computation instead of discussions. Each individual maximizes his utility by choosing the rates of saving, gift, fertility and bequest. The representative individual solves the following maximization problem: (1) subject to:

$$C_t^t = W_t + b_tW_t - [s_t+\tau_p+\tilde{\tau}+g_t+h_t]W_t, \quad (50)$$

$$C_{t+1}^{t} + (1+n_t)b_{t+1}W_{t+1} = (1+r_{t+1})s_tW_t + (1+n_{at})\tau_pW_{t+1} + (1+n_t)\tilde{\tau}W_{t+1} + (1+n_t)g_{t+1}W_{t+1}. \quad (51)$$

The equilibrium solutions are:

$$(1+n)(1+\theta) = (1+r)\beta/\gamma, \quad (52)$$

$$s = \beta e/\gamma, \quad (53)$$

$$h = [1-e(\alpha+\beta)/\gamma-dh_p(\alpha+\gamma)/\gamma]/[1+d(\alpha+\gamma)/\beta], \quad (54)$$

$$c_1 = 1-\beta e/\gamma-dh_p-(1+d\gamma/\beta)h, \quad (55)$$

$$\tau = d\gamma(h-\tilde{h})/\beta, \quad (56)$$

$$b = \sigma(e+dh_p+dh\gamma/\beta)/\beta, \quad (57)$$

$$g = d\gamma(h - \tilde{h}) / \beta + b \,, \tag{58}$$

where τ is the rate of net intertemporal transfer excess of $\tau_p + \tilde{\tau}$, or the rate of net intertemporal transfer outside of the PFLPPS. The total rate of net intertemporal transfer in the equilibrium of the PFLPPS is $\tau_p + \tilde{\tau} + \tau$. And $\tau_p + \tilde{\tau} + g$ is equivalent to the total gift rate in the equilibrium of the PFLPPS.

VII. Comparison of Different Equilibriums

Comparing the above equilibrium solutions gives:

$$\bar{n} = \hat{n} > \tilde{n} = n \,, \tag{59}$$

$$\bar{\theta} = \hat{\theta} < \tilde{\theta} = \theta \,, \tag{60}$$

$$\bar{h} = \hat{h} > \tilde{h} = h \,, \tag{61}$$

$$\bar{s} = \tilde{s} = \hat{s} = s \,, \tag{62}$$

$$\bar{c}_1 = \hat{c}_1 < \tilde{c}_1 = c_1 \,, \tag{63}$$

$$\bar{\tau} = \tau_p + \hat{\tau} < \tau_p + \tilde{\tau} = \tau_p + \tilde{\tau} + \tau \,, \tag{64}$$

$$\bar{b} = \hat{b} < \tilde{b} = b \,, \tag{65}$$

$$\bar{g} = \tau_p + \hat{g} < \tau_p + \tilde{g} = \tau_p + \tilde{\tau} + g \,, \tag{66}$$

$$\tau = 0 \,, \tag{67}$$

$$\hat{\tau} < 0 \,, \tag{68}$$

$$\bar{\tau} > \tilde{\tau} > \tau > \hat{\tau} \,, \tag{69}$$

$$\bar{g} > \tilde{g} > g > \hat{g} \,, \tag{70}$$

$$\tau_p > \bar{\tau} \,, \; s_p < \hat{s} \,, \tag{71}$$

$$h_p > \hat{h} \,, \; n_p > \hat{n} \,, \; \theta_p < \hat{\theta} \,, \text{ if } \beta \le \sqrt{\alpha\gamma + \gamma^2} \,, \text{ and} \tag{72}$$

$$c_p < \hat{c}_1 \,, \text{ if } \alpha > (1 - \beta^2) / \beta \,. \tag{73}$$

In the above maximization problems, the first-order condition $\alpha / C_t^t = (1 + r)\beta / C_{t+1}^t$ holds for the WPPS, the CPPS, the FFLPPS and the PFLPPS. If $c_{2t} = C_{t+1}^t / W_t$ is defined as the rate of retirement-period consumption, and $c_t = \left(C_t^t + C_{t+1}^t\right) / W_t$ as the lifetime consumption rate, then $c_{2t} = c_{1t}(1 + r)\beta / \alpha$ and $c_t = c_{1t}\left[1 + (1 + r)\beta / \alpha\right]$ holds for the four systems. Thus the behavior of work-period consumption rate can represent that of the retirement-period consumption rate and that of the lifetime consumption rate in the four systems. Of course, this result holds in equilibrium.

Equations (59)-(66) give the following result.

Result 1. The rates of fertility, output growth, child-rearing cost, saving, consumption, net intertemporal transfer, bequest and gift under the FFLPPS are the same as those under the WPPS. The rates of fertility, output growth, child-rearing cost, saving, consumption, net intertemporal transfer, bequest and gift under the PFLPPS are the same as those under the CPPS.

This result means that the fully-fertility-linked public pension system is equivalent to the system without public pension, and the partly-fertility-linked public pension system is equivalent to the conventional public pension system.

Equations (59) and (61) yield the following result.

Result 2. The rates of fertility and child-rearing cost under the CPPS (or PFLPPS) are smaller than those under the FFLPPS (or WPPS).

Some developed countries are suffering from low fertility. It seems that the FFLPPS may be useful to the developed countries. Zhang and Zhang (1995) make such a suggestion. However, it is necessary for this paper to explore further.

Equations (60), (63), (64), (65) and (66) give the following result.

Result 3. The rates of output growth, consumption, net intertemporal transfer, bequest and gift under the CPPS (or PFLPPS) are greater than those under the FFLPPS (or WPPS).

Equations (69), (67) and (68) give the following result.

Result 4. The rate of net intertemporal transfer under the WPPS is larger than the rate of net intertemporal transfer outside of the CPPS, both of which are positive. However, the rate of net intertemporal transfer outside of the PFLPPS is zero, and the rate of net intertemporal transfer outside of the FFLPPS is negative.

Equation (67) means that the gifts outside of the PFLPPS are exactly equal to the bequests outside of the PFLPPS. Equation (68) implies that the gifts outside of the FFLPPS are less than the bequests outside of the FFLPPS.

Equation (70) gives the following result.

Result 5. The gift rate under the WPPS is the largest. The gift rate outside of the public pension system under the CPPS is larger than that under the PFLPPS, which in turn is larger than that under the FFLPPS.

Equations (71), (72) and (73) give the following result.

Result 6. Comparing with the WPPS, the CPPS, the FFLPPS and the PFLPPS, the optimal allocation has the highest rate of net intertemporal transfer and the lowest saving rate; the highest rates of child-rearing cost and fertility and the lowest rate of output growth if $\beta \leq \sqrt{\alpha\gamma + \gamma^2}$ *; the lowest rate of work-period consumption if* $\alpha > (1-\beta^2)/\beta$ *.*

Some results of this paper are different from Nishimura and Zhang (1995) because their model is an exogenous model with one-sided altruism and without growth. Among the limited comparable results, the main differences are as follows:

- Difference 1. In Nishimura and Zhang (1995), the fertility rate under the WPPS is equal to that under the CPPS, which is larger than that under the FFLPPS. However, in this paper, the fertility rate under the FFLPPS (or WPPS) is larger than that under the CPPS.

- Difference 2. In Nishimura and Zhang (1995), the rate of net intertemporal transfer under the FFLPPS is larger than that under the CPPS and that under the WPPS. However, in this paper, the rate of net intertemporal transfer under the FFLPPS (or WPPS) is smaller than that under the CPPS (or PFLPPS).

- Difference 3. In Nishimura and Zhang (1995), the saving rate under the FFLPPS is larger than that under the WPPS, which in turn is larger than that under the CPPS. However, in this paper, they are identical.

- Difference 4. In Nishimura and Zhang (1995), the work-period consumption rate under the FFLPPS is larger than that under the WPPS. However, in this paper, they are equal to each other.

Some results in this paper are also different from Zhang and Zhang (1995) because the altruism is one-sided and the PFLPPS is not considered in their model. Among the limited comparable results, the main differences are as follows:

- Difference 1. In Zhang and Zhang (1995), the fertility rate under the WPPS is higher than that under the FFLPPS if $\gamma < \beta \leq \sqrt{\alpha\gamma + \gamma^2}$. However, in this paper, they are the same.

- Difference 2. In Zhang and Zhang (1995), the growth rate under the FFLPPS is higher than that under the WPPS if $\beta > \gamma$. However, they are the same in this paper.

- Difference 3. In Zhang and Zhang (1995), the optimal allocation has a higher growth rate than the WPPS if $\beta > \gamma$. However, in this paper, it is just the contrary if $\beta \leq \sqrt{\alpha\gamma + \gamma^2}$.

VIII. Conclusions

Under the overlapping generations model with two-sided altruism and endogenous growth, this paper compares the equilibrium solutions of the system without public pension (WPPS), the conventional public pension system (CPPS), the fully-fertility-linked public pension system (FFLPPS) and the partly-fertility-linked public pension system (PFLPPS). It proves that the fully-fertility-linked public pension system is equivalent to the system without public pension, and the partly-fertility-linked public pension system is equivalent to the conventional public pension system. This is an interesting finding that has not been revealed in the literature.

This paper shows that the rates of fertility and child-rearing cost under the CPPS (or PFLPPS) are smaller than those under the FFLPPS (or WPPS). Zhang and Zhang (1995) also obtains that the fertility rate under the CPPS is lower than that under the WPPS. The result, the fertility rate under the CPPS is lower than that under the FFLPPS, is different from that of Nishimura and Zhang (1995).

This paper also indicates that the rates of output growth, consumption, net intertemporal transfer, bequest and gift under the CPPS (or PFLPPS) are greater than those under the FFLPPS (or WPPS). Some comparison results concerning these rates are different from those in Nishimura and Zhang (1995) and Zhang and Zhang (1995). The differences have been shown in the last section.

The above main results have valuable policy implications. For example, the CPPS has a relative advantage to the PFLPPS based on the equivalence of the two systems. The compulsory public pension tax rate under the CPPS is lower than that under the PFLPPS. Therefore the CPPS is easier to carry out than the PFLPPS, especially for the developing countries.

The FFLPPS has a relative advantage to the WPPS based on the equivalence of the two systems. One of the basic functions of a public pension system is to prevent individual myopia. The FFLPPS has the function, while the WPPS has not. It is almost impossible for any country that has established a public pension system to abolish it.

It is better for the developing countries that have not established public pension systems to introduce the CPPS because it can promote economic growth, restrain population explosion, reduce the child-rearing cost rate, and increase the consumption rate. Zhang and Zhang (1995) make the same suggestion based on the rates of fertility and output growth.

It is necessary for developed countries to weigh gains and losses if they hope to transform their CPPS or PFLPPS[30] to the FFLPPS. Although the FFLPPS can increase the fertility rate, it may decrease the economic growth rate, increase the child-rearing cost rate, reduce the consumption rate, etc. Thus it is better for a developed country to transform its CPPS (or PFLPPS) to the FFLPPS if it cares more about the problem of population ageing and low fertility than the others. Otherwise it is suitable to maintain the CPPS (or PFLPPS). This is different from Zhang and Zhang (1995).

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[30] Nishimura and Zhang (1995, p.187) report that some developed countries have the PFLPPS.